PASSIVE IDENTIFICATION OF ACOUSTIC PROPAGATION MEDIA WITH VISCOUS DAMPING

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ABSTRACT

This paper concerns passive identification of an acoustic propagation medium with viscous damping. We propose a Green correlation approach to retrieve time of arrival between two sensors and to retrieve also times of arrival of first echoes. Green correlation is introduced as the cross-correlation of acoustic field generated by a white noise exciting the medium. This approach is experimentally validated and compared to the classical method which consists in retrieving the Green function from the Green correlation through a Ward identity. We show that this latter approach appears to be numerically inaccurate and above all unnecessary in the acoustic case with viscous damping.

1. GENERAL INFORMATION

Passive identification of a propagation medium consists in retrieving medium parameters, by using uncontrolled noise fluctuations only [14]. Such an idea has long been pursued in acoustics [11], [14] and seismology [4], [15] and gave rise to numerous applications and experimental validations [13], [7].

Preceding studies [11], [4], [13], [7] rely upon the estimation of the Green function of the medium. Such estimation is made possible by exploiting the celebrated Ward identity [16], which relates the noise correlation function to the Green function [11], [14], [6]. The fundamental role of dissipation in Ward identity was outlined in [7], were dissipation is assumed to be constant ; however, a constant dissipation model is hardly acceptable from a physical point of view [9], [12], and needs to be further discussed.

The contribution of the paper is to study and to validate experimentally a Green correlation approach in passive identification of acoustic media with viscous damping. Green correlation is introduced as the correlation of propagated white noise fields [8]. The motivation for introducing Green correlation comes from classical system identification theory, where noise based identification relies strongly on the transformation of second order statistics through linear systems [10].

To interpret acoustic Green correlation in bounded media, we derive expressions for the acoustic Green function and we relate them to the Green correlation by deriving Ward identities. Acoustic Green function is derived using Fourier theory. Low attenuation case is considered as it allows to obtain an explicit expression of the Green function and an explicit Ward identity in the temporal domain.

Those results are compared theoretically to classical acoustic results where constant damping model is considered [8], [7], [6]. Secondly, results are experimentally validated by showing that Green correlation allows to retrieve time of arrival between two sensors and also time of arrival of first echoes. We compared this approach to classical method which consists to retrieve the Green function from the estimated Green correlation and a Ward identity.

The organization of the paper is the following. In **section 2**, we introduce passive identification through a linear systems approach. We recall the definition of the Green function and its role in medium identification. Cross-correlation of random fields is recalled, and white noise notion is introduced. Then, we defined the Green correlation and we show its role in passive identification.

In **section 3**, acoustic waves equation with viscous damping is recalled. The acoustic Green function is computed for unbounded and bounded media. Low attenuation case is also considered.

Acoustic Green correlation is computed in **section 4**. Role of Green correlation in passive identification is emphasized. Ward identities are derived and compared to existing one for a constant damping model. Approximated Ward identities are derived in the low attenuation case.

In **section 5**, we show through an experimental validation that the Green correlation is a powerful tool in practical situation, especially when theoretical Ward identity can not be used due to signals digitalisation letting the Green function estimation unusable.

2. GREEN FUNCTION AND GREEN CORRELATION OF A LINEAR PROPAGATION MEDIUM : DEFINITION AND ROLE IN MEDIUM IDENTIFICATION

2.1 Green function of a linear medium.

We denote by $\mathbf{u}(t,\underline{x})$ the value of field $\mathbf{u}$ at time t and position $\underline{x}$. A time-shift invariant linear propagation medium X satisfies the superposition theorem *i.e.* the value $\mathbf{u}(t,\underline{x})$ of the generated field can be seen as the superposition of all contributions of elementary sources $\mathbf{f}(t',\underline{x}')dt'd\underline{x}'$ emitted at time t' during dt' period in the volume centered in $\underline{x}'$ and of dimensions $d\underline{x}'$, for all times t' and points x'. Mathematically, this can be written $\mathbf{u}(t,\underline{x}) = \int_{\mathbf{R}\times X} \mathbf{G}(t-t',\underline{x},\underline{x}')\mathbf{f}(t',\underline{x}')dt'd\underline{x}'$ and simplified by introducing the generalised convolution $\otimes_{T,S}$ as:

$$\mathbf{u} = \mathbf{G} \otimes_{T,S} \mathbf{f} \tag{1}$$

$\mathbf{G}$ is the Green function of the medium. All medium parameters are contained in its expressions. This highlights the importance of retrieving the Green function in medium identification.

Physically, $\mathbf{G}$ corresponds to the medium response of a spatio-temporal impulsion. According to this interpretation, $\mathbf{G}$ is sometimes called "impulse response" of the medium in reference to the classical impulse response of a linear system [10]. In that system approach, $\mathbf{f}$ is the input and the generated field $\mathbf{u}$ is the output of the system.

2.2 Cross-correlation of stochastic fields.

In passive identification, source fields are not controlled. The principle relies on recording noise sources and using their statistical properties to retrieve medium parameters. With stochastic source fields, the analysis has to be performed from the cross-correlation of the generated field $\mathbf{u}$ defined as:

$$\mathbf{C}_{\mathbf{u}}(t,\underline{x},\underline{x}') = \lim_{\tau \to +\infty} \frac{1}{\tau} \int_0^{\tau} \mathbf{u}(t+t',\underline{x})\mathbf{u}(t',\underline{x}')dt' \tag{2}$$

This formula is true for stationary and ergodic fields. This assumption is not a strong constraint in practice.

2.3 White noise.

By definition, a white noise is a field which value of a given time and position is uncorrelated to any other value taken at all other times and positions. Mathematically, the cross-correlation of a such field $\mathbf{f}$ is a spatio-temporal impulsion:

$$\mathbf{C}_{\mathbf{f}}(t,\underline{x},\underline{x}') = \delta(t)\delta(\underline{x},\underline{x}') \tag{3}$$

where δ is the Dirac distribution.

A white noise has no physical reality because it has an infinite power. However, in practice the temporal whiteness is only needed in a limited frequency band. This latter is defined by the used instrumentation. The classical approach [5] to justify that ambient noise converges to a source with a spatial whiteness consists to see the medium as a chaotic dynamical system. Then, according to equipartition theorem, it exists a time after which a coherent source snared in the medium becomes spatially white. This time, called mixing time, depends on the frequency band, medium geometry and heterogeneity. With those considerations, cross-correlation of ambient noise is stacked during a sufficient long time in order to obtain a contribution of an approximated white noise [11], [4], [7].

2.4 Green correlation.

As the Green function is the field generated by a spatio-temporal impulsion source *i.e.* $\mathbf{f}(t,\underline{x}) = \delta(t,t')\delta(\underline{x},\underline{x}')$, we define by analogy the Green correlation $\mathbf{C}$ by the cross-correlation of a field generated by a white noise source. This function, introduced in [8], plays by definition a fundamental role in passive identification.

We can precise Green correlation expression using definition (2) and equation (1). Indeed, for every generated field $\mathbf{u}$, $\mathbf{C}_{\mathbf{u}}$ can be expressed as:

$$\mathbf{C}_{\mathbf{u}} = \mathbf{G} \otimes_{T,S} \mathbf{G}^{-} \otimes_{T,S} \mathbf{C}_{\mathbf{f}} \tag{4}$$

where $\mathbf{G}^{-}(t,\underline{x},\underline{x}') := \mathbf{G}(-t,\underline{x},\underline{x}')$. It is important to note that to establish (4), we use the property: $\mathbf{G}(t,\underline{x},\underline{x}') = \mathbf{G}(t,\underline{x}',\underline{x})$, true for all times t and positions $\underline{x}$ according to spatial reciprocity. Equation (4) is the "order two" version of equation (1). When the source is a white noise, we obtain by using equation (4) an expression of the Green correlation:

$$\mathbf{C} := \mathbf{G} \otimes_{T,S} \mathbf{G}^{-} \tag{5}$$

This shows the fundamental importance of the Green correlation in medium identification when statistical properties of ambient sources are taking into account. Equations (4) and (5) show that "perfect" white noise is for passive identification what "perfect" impulsion is for active identification.

3. ACOUSTIC PROPAGATION WITH VISCOUS DAMPING : EQUATION AND GREEN FUNCTION.

3.1 Equation.

We consider an isotropic and homogeneous acoustic medium. Let v be the sound velocity and α be the viscous damping coefficient. Let $\mathbf{f}$ be a causal spatio-temporal pressure source and $\mathbf{u}$ the pressure field. The acoustic waves equation with viscous damping is then [12]:

$$\left[\frac{\partial^2}{\partial t^2} - \alpha^2 \frac{\partial}{\partial t}\Delta - v^2\Delta\right]\mathbf{u} = \mathbf{f} \tag{6}$$

where Δ is the Laplacian operator. To well-define this equation we have to add fields causality assumption and boundary conditions.

3.2 Green function in an unbounded medium.

For an unbounded medium, the acoustic Green function is space-shift invariant *i.e.* we can do the following substitution: $\mathbf{G}(t,\underline{x},\underline{x}') \leftrightarrow \mathbf{G}(t,\underline{x}-\underline{x}')$. The case of a bounded medium is discussed in next sub-section.

We denoted by

$$\widehat{\mathbf{G}}(\omega,\underline{k}) := \int_{\mathbf{R}\times\mathbf{R}^3} \mathbf{G}(t,\underline{x})e^{-\mathrm{i}\left(\omega t - \underline{k}^T \underline{x}\right)}dtd\underline{x} \tag{7}$$

the Fourier transform of $\mathbf{G}$ in the $(\omega,\underline{k})$-domain where T transpose, and, where ω and $\underline{k}$ are the frequency variables associated with t and $\underline{x}$, respectively.

As the Green function is the response to a spatio-temporal impulsion source *i.e* $\mathbf{f} := \delta(t,\underline{x})$, we obtain from equation (1):

$$\hat{\mathbf{G}}(\omega,\underline{k}) = \frac{\omega^{-2}k^2(\omega)}{k^2(\omega)-k^2} \tag{8}$$

where:

$$k^2(\omega) := \frac{\omega^2}{\mathbf{v}^2(\omega)} \tag{9}$$

with: $\mathbf{v}^2(\omega) := v^2 + \mathbf{i}\omega\alpha^2$, $k^2 := \underline{k}^T\,\underline{k}$. $k(\omega)$ is the relation of dispersion of acoustic waves with viscous damping. When low attenuation is considered *i.e.* $\omega\alpha^2/v^2 << 1$, we get:

$$k(\omega) \approx \frac{\omega}{v} \tag{10}$$

This approximation of $k(\omega)$ will be useful to derive expression of the Green function and correlation and also Ward identities in the $(t,\underline{x})$-domain.

We compute now the elastic Green function in the $(\omega,\underline{x})$-domain. In that domain, fields are capped by a $\vee$. Applying inverse Fourier transform to equation (8) with respect to $\underline{k}$, we show using residue theorem [2] that:

$$\check{\mathbf{G}}(\omega,\underline{x}) = \frac{e^{-\mathbf{i}||\underline{x}||k(\omega)}}{4\pi||\underline{x}||\mathbf{v}^2(\omega)} \approx \frac{e^{-\mathbf{i}\omega\frac{||\underline{x}||}{v}}}{4\pi||\underline{x}||v^2} \tag{11}$$

The approximation holds for a low viscous damping and allows to retrieve the classical Green function when there is no damping. It gives an easy interpretable expression of the Green function as a pure phase which provides an information on $||\underline{x}||/v$ easy to extract.

Computation of a general expression for $\mathbf{G}(t,\underline{x})$ from equation (11) when dissipation occurs, is difficult. In the low attenuation case, we obtain from (11):

$$\mathbf{G}(t,\underline{x}) \approx \frac{\delta(t-||\underline{x}||/v)}{4\pi v^2||\underline{x}||} \tag{12}$$

This show that retrieving (12) allows to estimate the time of arrival $||\underline{x}||/v$.

3.3 Green function in a bounded medium with a low viscous damping.

In a practical context, the medium is generally bounded. From equation (12) and Descartes' laws, we can decomposed the acoustic Green function in a sum of attenuated pure delays *i.e.*:

$$\mathbf{G}(t,\underline{x},\underline{x}') = \sum_{n=0}^{+\infty} a_n(\underline{x},\underline{x}')\delta(t-t_n(\underline{x},\underline{x}')) \tag{13}$$

where $a_0(\underline{x},\underline{x}') := 1/(4\pi||\underline{x}-\underline{x}'||)$, $t_0 := ||\underline{x}-\underline{x}'||/v$, and, $a_n(\underline{x},\underline{x}')$ and $t_n(\underline{x},\underline{x}')$ are attenuation coefficients and echoes time of arrival, respectively. In practice, the sum appearing in (13) is a finite sum due to the decreasing of attenuation coefficients and because of quantification which provides only a finite set of values for fields recorded.

4. ACOUSTIC GREEN CORRELATION AND WARD IDENTITIES.

4.1 Unbounded case.

As the Green function is space-shift invariant in an unbounded medium, the Green correlation satisfies also this property according to (5). Then, we can do the following substitution: $\mathbf{C}(t,\underline{x},\underline{x}') \leftrightarrow \mathbf{C}(t,\underline{x}-\underline{x}')$.

We can compute the acoustic Green correlation in the $(\omega,\underline{k})$-domain. Applying the Fourier transform to equation (5) and noting that $\widehat{\mathbf{G}^-} = \widehat{\mathbf{G}}^\dagger$ where $\dagger$ conjugate, we obtain:

$$\hat{\mathbf{C}}(\omega,\underline{k}) = \hat{\mathbf{G}}(\omega,\underline{k})\hat{\mathbf{G}}(\omega,\underline{k})^\dagger \tag{14}$$

We can deduce a Ward identity in the $(\omega,\underline{k})$-domain from equation (14) and the following easy to prove lemma:

Lemma. *Let* $\mathbf{a}$ *be a non real complex. Then we have:* $\mathbf{a}\mathbf{a}^\dagger = -\left(\mathrm{Im}\,\mathbf{a}^{-1}\right)^{-1}\mathrm{Im}\,\mathbf{a}$.

Applying this lemma with $\mathbf{a} = \hat{\mathbf{G}}(\omega,\underline{k})$, we obtain:

$$\omega\hat{\mathbf{C}}(\omega,\underline{k}) = -\frac{1}{\alpha^2k^2}\mathrm{Im}\,\hat{\mathbf{G}}(\omega,\underline{k}) \tag{15}$$

In that domain, the acoustic Green correlation is proportional to the imaginary part of the Green function. The proportionally term $1/(\alpha^2k^2)$ is the inverse of the dispersion function of $\alpha^2\Delta$ which is the dissipation operator appearing in equation (6). From Ward identity (15), we can easily derived exact Ward identities in $(\omega,\underline{x})$- and $(t,\underline{x})$- domains:

$$\omega\check{\mathbf{C}}(\omega,\underline{x}) = -\alpha^{-2}\Delta^{-1}\mathrm{Im}\,\check{\mathbf{G}}(\omega,\underline{x}) \tag{16}$$

$$\frac{\partial\mathbf{C}(t,\underline{x})}{\partial t} = -\alpha^{-2}\Delta^{-1}\ \mathrm{Odd}\ \mathbf{G}(t,\underline{x}) \tag{17}$$

where $\mathrm{Odd}\,\mathbf{G} := 1/2\,(\mathbf{G}-\mathbf{G}^-)$ is the odd part of $\mathbf{G}$. Those identities extend classical ones for acoustic waves [11], [7], [6], [8] in a sense that the $\check{\mathbf{C}}$ is "proportional" to the imaginary part of $\check{\mathbf{G}}$ and the first-time derivative of $\mathbf{C}$ is "proportional" to the odd part of $\mathbf{G}$. However, when viscous damping is considered, the proportionality term is Δ^{-1} and it is not a constant operator. Then, (16) and (17) are not sufficiently explicit to interpret the relation existing between the Green function and the Green correlation. We get over this last step by considering a low viscous damping.

4.2 Unbounded and low viscous damping case.

First, we compute the acoustic Green correlation in the $(\omega,\underline{k})$-domain. Using Ward identity (15) and expression (8), we get:

$$\hat{\mathbf{C}}(\omega,\underline{k}) = \frac{\omega^{-4}|k(\omega)|^4}{|k^2(\omega)-k^2|^2} \tag{18}$$

Expression (18) proves that retrieving the Green correlation is sufficient to estimate medium parameters: v and α. Indeed, those parameters are contained in $k^2(\omega)$ which is the pole of $\hat{\mathbf{C}}(\omega,\underline{k})$. Inverse Fourier transform of equation (18) with respect to k and residue theorem give:

$$\check{\mathbf{C}}(\omega,\underline{x}) = -\frac{\mathrm{Im}\,e^{-\mathbf{i}||\underline{x}||k(\omega)}}{4\pi||\underline{x}||\omega^3\alpha^2} \approx \frac{\sin\left(\omega\frac{||\underline{x}||}{v}\right)}{4\pi||\underline{x}||\omega^3\alpha^2} \tag{19}$$

This expression shows that we can extract all the physical parameters from $\check{\mathbf{C}}$. Using relations (11) and (19), we obtain the following approximated Ward identities:

$$\omega^3\check{\mathbf{C}}(\omega,\underline{x}) \approx -\frac{v^2}{\alpha^2}\ \mathrm{Im}\ \check{\mathbf{G}}(\omega,\underline{x}) \tag{20}$$

$$\frac{\partial^3\mathbf{C}(t,\underline{x})}{\partial t^3} \approx -\frac{v^2}{\alpha^2}\ \mathrm{Odd}\ \mathbf{G}(t,\underline{x}) \tag{21}$$

(21) was deduced from (20). For a viscous damping and low attenuation framework, it is the third-time derivative which is directly proportional to the odd part of the Green function in the $(t,\underline{x})$-domain. The difference derivative order comes only from the form of the dissipation.

4.3 Bounded and low viscous damping case.

We show now how Green correlation can be useful in passive identification of bounded media. It is difficult to obtain an explicit expression of the Green correlation from equations (5) and (13). However, Ward identity (21) can be extended to any bounded media as:

$$\frac{\partial^3 \mathbf{C}(t,\underline{x},\underline{x}')}{\partial t^3} \approx -\frac{v^2}{\alpha^2}\,\mathrm{Odd}\;\mathbf{G}(t,\underline{x},\underline{x}') \tag{22}$$

Indeed, only the propagation equation (6) intervenes in computation of Ward identities. Boundary conditions only intervenes in Green function and Green correlation expressions. The equation (22) is important as it proves that times of arrival appearing in the Green function also appear, symmetrically with respect to time origin and with an opposite amplitude, in the Green correlation. This justifies that Green correlation estimation is theoretically sufficient to retrieve times of arrival of direct and reflected waves.

5. EXPERIMENTAL VALIDATIONS.

We consider two microphones, denoted by m_1 and m_2 measuring pressure field in the 2 Hz-17 kHz frequency band. Every measure is sampled at 51.2 kHz and high-pass filtered at 200 Hz in order to suppress electrical noises. Sensors are disposed in an acoustic air room and spaced of 26.5 cm. The experimental protocol relies on two steps: to retrieve the Green function by active identification and to retrieve the Green correlation by passive identification. Then, results are discussed and compared.

In the first step, m_1 is seen as a source and m_2 as a receiver. Indeed, an acoustic signal is emitted from a speaker closely to sensor m_1. Five seconds of signals recorded by m_1 and m_2 are used to retrieve the Green function of the medium between the two sensors. We use a classical recursive least square algorithm [1] to retrieve the finite impulsion response filter (1024 points, 20 ms) representing numerically the Green function. The convergence really appears after 500 ms of signals recorded.

In the second step, m_1 and m_2 are both seen as receivers. Speakers emitting temporal white noise, generated numerically, where displaced around the two sensors in order to "simulate" a spatio-temporal white noise. Signals received by m_1 and m_2 where used to compute cross-correlations on 20 ms. All correlations where stacked during 5 s to obtain the estimated Green correlation. The convergence really appears after 1 s.

Odd part of the estimated Green function, and, the estimated Green correlation are represented and compared in Figure 1. We normalised each estimated field by dividing by their maximum in order to not take their amplitude into account.

The estimated Green function contains its first significant pic at 0.74 ms which corresponds to a speed sound of $v = 348$ m/s which is an acceptable value at 22°C [3].

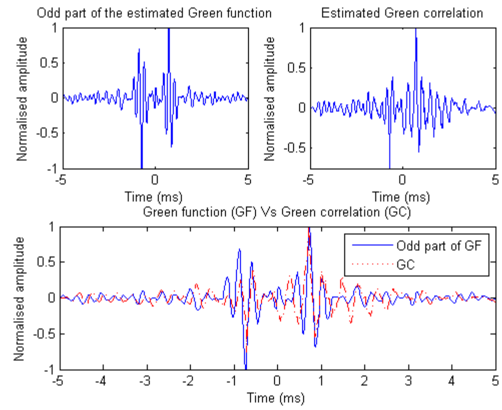

Figure 1: Representation of the estimated Green function and the estimated Green correlation. The two functions are compared in last sub-figure.

Furthermore, we observe echoes justified by the reflections on room interfaces, this valid the model proposed by equation (13). We observe a similar evolution for the estimated Green correlation. There is not a perfect symmetry of the Green correlation due to the imperfect symmetry of ambient sources distribution used. We retrieve the time of arrival at ± 0.74 ms with an opposite amplitude for positive and negative lags. First echoes match with the odd part of the Green function. Then, for lag t such $|t| > 1$ ms echoes do not really correspond between Green function ones and Green correlation ones.

We can conclude on this experimentation that we retrieve perfectly time of arrival of the direct path. We observe a drift on times of arrival of echoes. This is not a problem to identify sound velocity or distance sensor-sensor. However, geometrical identification of the medium from time of arrival of echoes appearing in the Green correlation is clearly less accurate. This result can also be observed in [11].

We compare in Figure 2 the odd part of the Green function with the first-time derivative of the Green correlation, and, with the third-time derivative of the Green correlation. We see, naturally, that the more the time derivative order increases the more the signal to noise ratio decreases and times of arrival estimation becomes difficult. This proves that Ward identity is an important theoretical tool to compute and interpret the Green correlation from the Green function but it is difficult to apply numerically and above all unnecessary.

6. CONCLUSION.

We applied Green correlation theory to acoustic waves with viscous damping. We computed, in useful representation domains, the Green function, the Green correlation and Ward identities associated. We considered low attenuation case, which provides interesting and interpretable Ward identities in $(\omega,\underline{x})$- and $(t,\underline{x})$-domains. More precisely, in this latter domain, it is the third-time derivative of the elastic Green correlation which is proportional to the odd part of the elastic Green function. This is a direct consequence of viscous damping model and this result was validated experimentally and compared to classical ones in acoustic with a constant

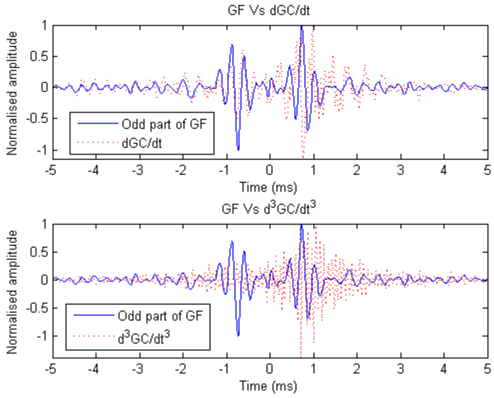

Figure 2: Comparison of the odd part of the Green function with the first-time derivative of the Green correlation, and with the third-time derivative of the Green correlation.

damping model. We prove that retrieving the Green correlation is sufficient to estimate the direct time of arrival and first times of arrival of echoes (before 1 ms for our experimentation). This confirms that Green correlation is an interesting practical tool in passive identification.

REFERENCES

[1] B.D.O. Anderson, J.B. Moore, *Optimal Filtering*, Dover Publications, 2005.

[2] G.B. Arfken, H.J. Weber, *Mathematical Methods for Physicists*, Elsevier, 6th edition, 2005.

[3] D.A. Bohn, *Environmental Effects on the Speed of Sound*, J. Audio Eng. Soc. **36**, 1988, No. 4.

[4] M. Campillo, A. Paul *Long-Range Correlations in the diffuse seismic coda*, Science **299**, 2003, p. 547-549.

[5] M. Campillo, *Phase and Correlation in "Random" Seismic Fields and the Reconstruction of the Green Function*, Pure and Applied Geophysics **163**, 2006, p. 475-502.

[6] Y. Colin de Verdiere, *Semi-classical analysis and passive imaging*, NonLinearity **22**, 2009, R45-R75.

[7] P. Gouedard *and al.*, *Cross-correlation of random fields: mathematical approach and applications*, Geophysical Prospecting **56**, 2008, p. 375-393.

[8] J.L. Lacoume, *Tomographie passive: observer avec du bruit*, Colloque GRETSI, 2007.

[9] L. Landau, E. Lifchitz, A. Kosevich *Physique thorique : thorie de l'lasticit*, 2nd edition, Editions MIR, 1990.

[10] L. Ljung, *System Identification: Theory for the User*, Prentice Hall, 2nd edition, 1999.

[11] O. Lobkis, R. Weaver, *On the emergence of the Green function in the correlations of a diffuse field*, JASA *110*, 2001, p. 3011-3017.

[12] D. Royer, E. Dieulesaint, *Elastic Waves in Solids*, Springer, 2000.

[13] K.G. Sabra *and al.*, *Extracting time-domain Green function estimates from ambient seismic noise*, Journal of Ocean Engineering **30**, 2005, p. 338-347.

[14] R. Snieder, K. Wapenaar, U. Wegler, *Unified Green function retrieval by cross-correlation; connection with energy principles*, Physical Review E **75**, 2008, 14 p.

[15] K. Wapenaar, E. Slob, R. Snieder, and A. Curtis, 2010, *Tutorial on seismic interferometry. Part II: Underlying theory and new advances*, Geophysics **75**, 2010, *in press*.

[16] R. Weaver, *Ward identities and the retrieval of Green's functions in the correlations of a diffuse field*,Wave Motion **45**, 2008, p. 596-604.